2.1 Product and Quotient Rules: Form A Assessment

Name: _____ Time: 60 minutes No calculator 100 points

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Answer exactly the question asked. Show work on every question except Question 11. Angles are in radians. If a derivative does not exist, write DNE. If a function value does not exist, write *undefined*.

1. Find $f'(x)$ for each function. **(3 each, 24 pts)**
   You need not list x-values where $f'(x)$ does not exist.

   (a) $f(x) = x^4 \sin x$

   (b) $f(x) = (2x-3)(x^2+4x)$

   (c) $f(x) = \dfrac{x^2-1}{x^2+1}$

   (d) $f(x) = \dfrac{x^3-4x}{\sqrt{x}}$

   (e) $f(x) = \dfrac{12x^2+8x}{3x^2+2x}$

   (f) $f(x) = \sin x \csc x + 5\cot x$

   (g) $f(x) = x \sec x$

   (h) $f(x) = \dfrac{\cot\left(\frac{\pi}{4}\right)}{\csc\left(\frac{\pi}{6}\right)}$

2. Let $f(x) = \dfrac{\sin x}{1-\cos x}$. Find $f'\left(\frac{\pi}{3}\right)$. **(5 pts)**

3. Evaluate each limit. **(3 each, 6 pts)**

   (a) $\lim\limits_{h\to 0} \dfrac{\tan\left(\frac{\pi}{4}+h\right)-1}{h}$

   (b) $\lim\limits_{x\to\pi} \dfrac{\sec x + 1}{x-\pi}$

4. Let $f(x) = \dfrac{x^2}{x+1}$. Find an equation of the tangent line to the graph of f at $x = 1$. **(6 pts)**

5. Let $f(x) = x \tan x$. Find the y-intercept of the line tangent to the graph of f at $x = \frac{\pi}{4}$. **(5 pts)**

6. Let $f(x) = \sin x \cos x$. **(6 pts)**
Find every x in the interval $(0, \pi)$ at which the graph of f has a horizontal tangent line.

7. Derive the rule for the derivative of $\sec x$. **(6 pts)**
Write $\sec x = \dfrac{1}{\cos x}$ and use the quotient rule to show that the derivative of $\sec x$ is $\sec x \tan x$. Quoting the rule for $\sec x$ earns no credit.

8. The table gives selected values of f, g, and their derivatives. **(4 each, 12 pts)**

Let $h(x) = f(x)\,g(x)$, $k(x) = \dfrac{f(x)}{g(x)}$, and $m(x) = f(2)\,g(x)$.

x	$g(x)$	$f'(x)$	$g'(x)$	$f(x)$
0	-2	3	4	1
2	1	2	-2	-3
4	-4	-1	3	2

(a) Find $h'(2)$.

(b) Find $k'(4)$.

(c) Find $m'(4)$.

9. Each part describes a different function. **(4 each, 12 pts)**

(a) Let $h(x) = f(x)\,g(x)$. It is known that $f(1) = 2$, $f'(1) = -3$, $g(1) = 4$, and $h'(1) = -6$. Find $g'(1)$.

(b) Let $q(x) = \dfrac{f(x)}{g(x)}$. It is known that $f(3) = 6$, $g(3) = 2$, $g'(3) = 1$, and $q'(3) = -1$. Find $f'(3)$.

(c) Let $w(x) = x\,f(x)$. It is known that $f'(2) = 3$ and $w'(2) = 4$. Find $f(2)$.

10. Figure A10 shows the graphs of f (solid) and g (dashed). **(4 each, 12 pts)**
Each graph is made of straight line segments that meet at grid points. Let $p(x) = f(x)\,g(x)$, $q(x) = \dfrac{f(x)}{g(x)}$, and $r(x) = x^2 f(x)$.

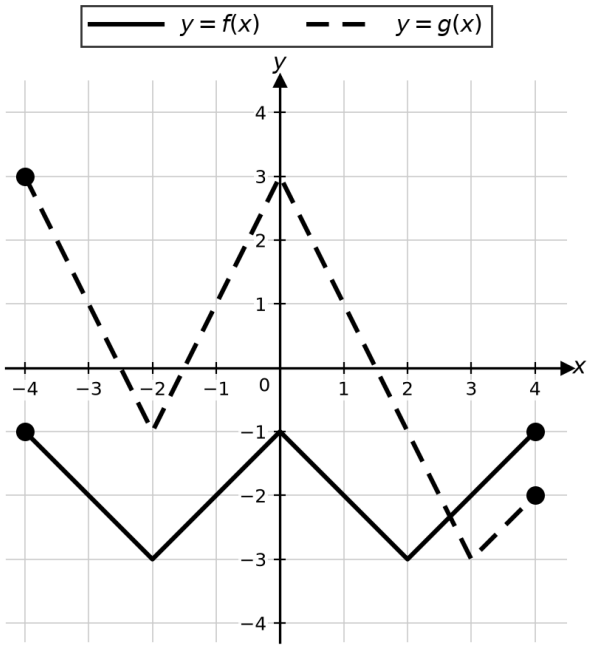


Figure A10

(a) Find $p'(-1)$.

(b) Find $q'(1)$.

(c) Find $r'(-3)$.

11. Each graph below shows two curves on the same axes. **(6 pts)**
In exactly one of the graphs, one curve is the derivative of the other. Circle the Roman numeral of that graph.

